Sub.Code : 216

NEB-GRADE XII

Mathematics

It is for those students whose first two digit of registration number starts from 68 to 77. Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Time: 3 hrs.

Full Marks: 100

Pass Marks:35 (For partial students only)

Note: Group A is compulsory and select another one Group either B or C. Group 'A'

Attempt all the questions.

- a) In how many ways the letters of the word "CURIOUS" can be arranged so that all the vowels are always together ? (2)
 - b) Find the term independent of x in the binomial expansion of

$$\left(2x + \frac{1}{2x}\right)^{10} \tag{2}$$

- c) Prepare a Caley's table for $G = \{1, \omega, \omega^2\}$ where ω is the cube root of unity under multiplication. (2)
- 2. a) Determine the equation of the hyperbola with vertex (8, 0) and passing through $(8\sqrt{2}, 4)$ (2)
 - b) Find the angle between the lines whose direction cosines are proportional to 1, 2, 2 and 2, 3, 6 (2)
 - c) Given $\vec{a} = 3\hat{i} + 2\hat{j}$, and $\vec{b} = -\hat{i} + 2\hat{j}$, find $\vec{a} + \vec{b}$ and its magnitude. (2)
- 3. a) Find the derivative of $Sinh^{-1}(Cosh x)$. (2)

b) Evaluate:
$$\int \frac{dx}{x + \sqrt{x^2 - 1}}$$
 (2)

c) Find the equation of normal to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4) (2)

4. a) Solve:
$$\sqrt{1-x^2} \, dy + \sqrt{1-y^2} \, dx = 0$$
 (2)

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- b) Find the mean deviation from mean of the data: 25, 30, 32, 38, 35 (2)
- c) Two dice are rolled simultaneously. Determine the probability of turning up the number whose sum is less than six. (2)
- 5. a) In how many ways a committee of three person can be formed out of 3 men and 4 women so as to include atleast one woman. (4)
 - b) Show that $\{2^{n+1}: n \in Z\}$ is an abelian group with respect to multiplication. (4)

Or

Define abelian group. If (G, *) is an abelian group, prove that

$$(a * b) - a = a - a * b - a \forall a, b \in G.$$

6. a) Find equation and the point of contact of tangent to the parabola $y^2 = 12x$ which makes an angle 45^0 with the straight line x - 2y + 3 = 0 (4)

Or

Find the eccentricity and coordinates of foci of $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$

b) Find the equation of the plane through the point (2, 2, 1) and (9, 3, 6) and normal to the plane 2x + 6y + 6z = 9 (4)

7. a) Evaluate:
$$\int \frac{dx}{4 + 3Coshx}$$
. (4)

Evaluate : $\int \frac{5}{(x+5)(2x^2+5)} dx$.

b) Solve:
$$\cos^2 x \frac{dy}{dx} + y = 1$$
 (4)

8. a) Calculate **Karl Pearson's** coefficient of correlation from the following data. (4)

Х	12	9	8	10	13	7
Y	14	8	6	9	12	3

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- (3)
- b) A certain manufacturing plant produces electric fuses of which 20% are defective. Find the probability that in a sample of 8 fuses selected at random there will be atleast one defective and not more that one defective.
- 9. Define exponetial and logarithm series. Also sum to infinity the series:

$$1^{2} + \frac{2^{2}}{2!} + \frac{3^{2}}{3!} + \dots, \dots$$
(6)

10. Define vector product of two vectors. If $\vec{a} + \vec{b} + \vec{c} = 0$, prove that

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$
 and interpret it geometrically. (6)

11. Find from definition, the derivative of *Sin (logx)*. (6)

Or

State **Rolle's theorem**, interpret it geometrically. Verify Rolle's theorem for f(x) = (x + 1) (x-2) in [-1, 2]

Group 'B'

- 12. a) Find the resultant and the angle subtended by it with P when the forces P and Q act at right angle. (2)
 - b) A body is projected vertically upwards with a velocity of $19.6ms^{-1}$. How long will it take to reach a point 294 m below the point of projection? $(g = 9.8 ms^{-2})$ (2)
 - c) A car covers a distance of 50m in 5 sec. against a frictional force. If the power of the engine is 400W, find the frictional force. (2)
- 13. a) Two forces of magnitude 3P, 2P respectively have a resultant R. If the first force be doubled, the magnitude of the resultant is doubled, find the angle between the forces. (4)

Or

Two forces P and Q acting parallel to the length and base of an inclined plane respectively, would each of them singly support a weight *W* on

the plane, prove that : $\frac{1}{P^2} - \frac{1}{Q^2} = \frac{1}{W^2}$.

b) A particle is projected up from the bottom of an inclined plane with a velocity of 25m/s, while another is dropped from the highest point to

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slide down the plane as the same moment. If the length of the plane be 200 m and the angle of inclination of the plane with the horizon is 30° , find when and where the two particles will meet. $(g = 10 \text{ m/s}^2)$ (4)

- 14. Define moment of a force. Also interpret it geometrically. State and prove Varignon's theorem for intersecting forces. (6)
- Define laws of motion. A gun of mass *l* metric tonne force a shot of mass 14kg and recoils up smooth inclined plane, rising to a height of 1.6 m, find the initial velocity of the projectile.

Or

A cannon ball has the same range R on a horizontal plane for two different angles of projection. If H and H^1 are the greatest heights and t_1 and t_2 are the time of flight in two paths for which this is possible, prove that:

a)
$$R^2 = 16HH^1$$
 b) $R = \frac{1}{2}gt_1t_2$

Group 'C'

- 16 a) Find the vertices of the feasible region determined by the constraints $3x + 2y \le 24$, $x + y \le 20$, $x \ge 0$, $y \ge 0$ (2)
 - b) Convert the binary number 10110010 to hexadecimal number. (2)
 - c) Write the condition for the system of equations (2) $a_{11}x_1 + a_{12}x_2 = b_1$, $a_{21}x_1 + a_{22}x_2 = b_2$ to be ill conditioned.
- 17. a) Solve the system of equations by Gauss elimination or Matrix inversion method: 2x 3y + z = 1, x 2y + 3z = 2, 3x-y+2z=9 (4)
 - b) Evaluate, using Simpson's rule: $\int_{0}^{1} \frac{dx}{1+x^{2}}, n = 4$ (4)
- 18. Using Simplex method, maximuze Z = 3x + 5y subject to constraints $3x + 2y \le 18$, $x \le 4$, $y \le 6$ and $x, y \ge 0$ (6)
- 19. Determine the number of positive roots and apply the method of successive bisection to find the roots of the equation $x^3 2x 5 = 0$ in (2, 3) correct to three places of decimals. (6)

Or

Using Newton Raphson method find a positive root of $x^3+3x - 5 = 0$ in (1, 2) correct to three places of decimals.